THE MOST COMPACT SCISSION CONFIGURATION OF FRAGMENTS FROM LOW ENERGY FISSION OF $^{234}\text{U}$ AND $^{236}\text{U}$

Modesto Montoya Zavaleta$^a$, $^b$

ABSTRACT

Using a time of flight technique, the maximal values of kinetic energy as a function of primary mass of fragments from low energy fission of $^{234}\text{U}$ and $^{236}\text{U}$ were measured by Signarbieux et al. From calculations of scission configurations, one can conclude that, for those two fissioning systems, the maximal value of total kinetic energy corresponding to fragmentations ($^{42}\text{Mo}_{62}$, $^{50}\text{Sn}_{80}$) and ($^{42}\text{Mo}_{64}$, $^{50}\text{Sn}_{80}$) respectively, are equal to the available energies, and their scission configurations are composed by a spherical heavy fragment and a prolate light fragment both in their ground state.

Keywords: Low energy fission; $^{234}\text{U}$; $^{236}\text{U}$; fragment kinetic energy; cold fission

RESUMEN

Usando una técnica de tiempo de vuelo, Signarbieux et al. midieron el valor máximo de la energía cinética total en función de la masa primaria de los fragmentos de la fisión de baja energía de $^{234}\text{U}$ y $^{236}\text{U}$. De los cálculos de las configuraciones de escisión, puede concluirse que, para esos dos sistemas fisiles, el valor máximo de la energía cinética corresponde a las fragmentaciones ($^{42}\text{Mo}_{62}$, $^{50}\text{Sn}_{80}$) y ($^{42}\text{Mo}_{64}$, $^{50}\text{Sn}_{80}$), respectivamente, son iguales a los valores disponibles de energía, y sus configuraciones de escisión están compuestas por un fragmento pesado esférico y un fragmento liviano prolatado, ambos en sus estados fundamentales.

PALABRAS CLAVE: Fisión a baja energía; $^{234}\text{U}$; $^{236}\text{U}$; energía cinética de fragmentos; fisión fría

INTRODUCTION

Since the discovery of the neutron–induced fission, this process was described by collective variables (deformation, vibration, rotation, etc.) and intrinsic variables (quasi-particles excitations). Nevertheless, the dynamic of fission process are not yet completely understood. In particular, it is neither known the nature of the coupling between the collective and intrinsic degrees of the freedom during the descend from the saddle to scission, nor known how it does arise.

THE MOST COMPACT SCISSION CONFIGURATIONS

In the process of thermal neutron induced fission of $^{233}\text{U}$, a composed nucleus $^{234}\text{U}$ with excitation energy equal to neutron separation energy ($B_n$) is formed first. Then, this nucleus splits in two complementary light and heavy fragments having $A_L$ and $A_H$ as mass numbers, and $E_L$ and $E_H$ as kinetic energies, respectively.

The Q-value of the this reaction is given by the relation

$$Q(Z,A;Z_2,A_2|Z_1,A_1) = M(Z_2,A_2) - M(Z_1,A_1) - M(Z,A)$$  \hspace{1cm} (1)

where $M(Z,A)$ is the mass of nucleus with $Z$ and $A$ as proton number and mass number, respectively.

This available energy at scission configuration is spend in pre-scission total kinetic energy ($TKE_0$), fragments interaction Coulomb energy ($CE$) and total fragments deformation energy,

$$TDE = DE_L + DE_H$$  \hspace{1cm} (2)

where $DE_L$ and $DE_H$ are the light and heavy fragment deformation energy, respectively, and in fragments intrinsic excitation energy,

$$TXE = XE_L + XE_H$$  \hspace{1cm} (3)

where $XE_L$ and $XE_H$ are the light and heavy fragment intrinsic excitation energy, respectively.

Then the balance energy at scission configuration results

$$Q + B_n = TKE_0 + CE + TDE + DXE$$  \hspace{1cm} (4)

where $B_n$ is the neutron separation energy from the composed nucleus after the neutron is absorbed.

If there is no neutron emission, the light and heavy fragments reach the detectors with their primary kinetic energies equal to $KE_L$ and $KE_H$, respectively. The total primary fragments kinetic energy will be

$$TKE = KE_L + KE_H = TKE_0 + CE + Q + B_n - TDE - DXE$$  \hspace{1cm} (5)

The maximal value of total kinetic energy is reached when the sum of $TDE$ and $TDX$ is minimal, i.e.

$$\begin{align*}
TKE_{\text{max}} &= (TKE_0 + CE)_{\text{max}} = Q + B_n - (TDE - DXE)_{\text{min}} \\
CE_{\text{max}} &= Q + B_n
\end{align*}$$

The most compact scission configuration occurs when maximal value of coulomb energy is equal to the available energy, i.e.

$$CE_{\text{max}} = Q + B_n$$

$^a$Instituto Peruano de Energía Nuclear, Avda. Canadá 1470, San Borja, Lima, Perú and
$^b$Facultad de Ciencias, Universidad Nacional de Ingeniería, Av. Túpac Amaru, 210, Apartado 31-139, Lima, Perú
In this case, from Eq. 5 one obtains the relations

$$TKE_{\text{max}} = CE_{\text{max}} = Q + B_n$$

and

$$DE_{\text{min}} = 0, \quad DX_{\text{min}} = 0 \text{ and } TKE_0 = 0.$$  \hspace{1cm} (9)

Not always this situation is possible to occur. Nevertheless we can assume that for each mass fragmentation the maximal value of total kinetic energy is obtained for similar condition, i.e. \( TKE_0 = 0 \), \( TXE = 0 \) and \( TDE = TDE_{\text{min}} \).

In what follows, we shall assume that the total kinetic energy distribution of the fission fragments can be approximated by a Gaussian distribution \[9\]

**DEFORMATION ENERGY**

The total energy of a nucleus is calculated at first approximation by a liquid drop model type \( W \), using the mass formula of Myers and Swiatecki \[6\]. The shell correction \( \delta U \) is calculated by the Strutinsky's method \[7\], using Nilsson Hamiltonian \[8\]:

$$V_{\text{core}} = -\kappa \left\{ \hat{l} \cdot \hat{s} + \mu \left( \hat{l}^2 - \langle \hat{l}^2 \rangle_N \right) \right\}$$  \hspace{1cm} (10)

where \( \kappa \) and \( \mu \) are the Nilsson's constants.

The pairing correction is calculated using the BCS method \[9\].

Then, the relation for the total energy of the nucleus \( (Z,N) \) results:

$$DE(Z,N,D) = W(Z,N,D) - \delta U_{Z,N} + \delta U_{N} + \delta U_{Z} + \delta E_{N} + \delta E_{Z}$$  \hspace{1cm} (11)

where \( W(Z,N,D) \) is the energy of a nucleus \( (Z,N) \) having deformation \( D \), and \( W_{\text{S}}(Z,N) \) the energy in its spherical shape.

The constant of the harmonic oscillator was the suggested by Nilsson \[5\]:

$$\hbar \omega_{\text{h}} = 41A^{1/3}.$$  \hspace{1cm} (12)

As one said, the total fragments kinetic energy is close to the available energy for light and heavy complementary fragments with masses around \( A = 104 \) and \( A = 152 \), respectively. Let us relate this result to the deformation for nuclei in this mass neighborhood.

The energies of nuclei \(^{106-108}\text{Mo}\) and \(^{106-108}\text{Tc}\) as a function of their corresponding deformations \( (\varepsilon) \) are presented on Figs. 1 and 2, respectively. The assumed Nilsson's constants \[5\] for these nuclei are \( \kappa_N = 0.678, \kappa_P = 0.07, \mu_N = 0.33 \) and \( \mu_P = 0.35 \).

As we can see, those nuclei have a prolate shape with to \( \varepsilon = 0.3 \) in their ground state. If the fragment deformation changes from \( \varepsilon = 0 \) to \( \varepsilon = 0.3 \) the deformation energy will decreases by around 2 MeV, while a change from \( \varepsilon = 0.3 \) to \( \varepsilon = 0.4 \) increases of deformation energy by 4 MeV. This result suggests that these nuclei are prolate and soft between \( \varepsilon = 0 \) to \( \varepsilon = 0.3 \) and became stiff for higher prolate deformations.

The energy as a function of deformation for nuclei \(^{130-132}\text{Sn}\) are presented on Fig.3. The assumed Nilsson's constants for these nuclei are \( \kappa_N = 0.635, \kappa_P = 0.067, \mu_N = 0.43 \) and \( \mu_P = 0.54 \).

![Fig. 1. Deformation energy for nuclei \(^{106-108}\text{Mo}\) calculated by a drop liquid model with pairing and shell correction \[6\]. See text.](image-url)
Fig. 2. Deformation energy for nuclei $^{106-108}$Tc calculated by a drop liquid model with pairing and shell correction [6].

One can see that $^{130}$Sn is softer than $^{132}$Sn. For a deformation from $\varepsilon = 0$ to $\varepsilon = 0.2$, the nucleus $^{130}$Sn spends around 5 MeV while the nucleus $^{132}$Sn, for the same deformation, spends 10 MeV. The neutron number number $N = 82$ and proton numbers around $Z = 50$ correspond to spherical hard nuclei.

The above characteristics of light fragments, corresponding to masses from $A = 100$ to $A = 106$, and their complementary fragments, corresponding to mass from $A = 130$ to $A = 132$, makes possible that their maximal values of the total kinetic energy of complementary fragments $TKE$ are close to the available energy.

![Fig. 3](image3.png)

Fig. 3: Deformation energy for nuclei $^{130-132}$Sn calculated by a drop liquid model with pairing and shell correction [3]. See text.

Fig. 4. Equipotential curves for scission configuration of fragments $^{42}$Mo$_{52}$, $^{50}$Sn$_{80}$ as a function of their deformation. $\varepsilon_L$ and $\varepsilon_H$ are the light and heavy fragment deformation.

For the case of $^{233}$U(n$_{th}$, f), the total kinetic energy of the couple $^{42}$Mo$_{52}$, $^{50}$Sn$_{80}$ is almost equal to the available energy. This result means that the corresponding scission configuration is composed by fragments in their ground state. On the Fig. 4 we can see the several equipotential energy of the scission configuration composed by those fragments given by the relation

$$V = CE + DE_H + DE_L$$  \hspace{1cm} (13)

where $\varepsilon_H$ and $\varepsilon_L$ are the heavy and light fragment deformation energy, respectively, calculated using the Nilsson model [5] and $CE$ is the interaction Coulomb energy between the two fragments separated by 2 fm. On this curve one obtains that for $\varepsilon_H = 0$ and $\varepsilon_L = 0.3$ the Coulomb energy is equal to the available energy to 204 MeV.
The results are similar to complementary fragments corresponding to the deformed transitional nuclei with $A_L$ between 100 and 106 (\(N\) between 60 \& 64) and to the spherical nuclei with $A_{II}$ around 132 (\(Z = 50\) and \(N = 82\)).

For the complementary fragments $^{42}$Mo$_{62}$ and $^{50}$Sn$_{80}$, the maximal value of CE corresponds to ground state nuclei or close to that. This case is unique. Other configurations will need deformation energy, which will be higher for the harder nuclei. On the Fig. 3 is presented the deformation energy for the spherical nuclei $^{130}$Se, $^{131}$Se and $^{132}$Se, respectively. We can see that the double magic nucleus $^{132}$Se need 2 MeV more than $^{130}$Se for going from the spherical state $\varepsilon = 0$ to the slightly deformed $\varepsilon = 0.05$. The fact that $^{132}$Se is not so hard as $^{132}$Se explain which the highest values of Coulomb interaction energy correspond to values close to the available energy for $^{233}$U(n$_{th}$,f) as well as for $^{235}$U(n$_{th}$,f).

CONCLUSION

From calculations of scission configurations from thermal neutron induced fission of $^{233}$U and $^{235}$U, one can conclude that the highest value of Coulomb interaction energy between complementary fragments correspond to fragmentations ($^{42}$Mo$_{62}$, $^{50}$Sn$_{80}$) and ($^{42}$Mo$_{64}$, $^{50}$Sn$_{80}$), respectively. For both cases the calculated maximal value of Coulomb interaction energy are equal to the available energy of the reaction for spherical (\(\varepsilon_{II} = 0\)) heavy fragments and prolate (\(\varepsilon_{II} = 0.3\)) complementary light fragments, which correspond to their ground states. Moreover the light fragments are soft between $\varepsilon_L = 0$ and $\varepsilon = 0.3$ and harder if they go to more prolate shapes; while the heavy fragment $^{50}$Sn$_{82}$ is no as hard as $^{50}$Sn$_{82}$. The calculated maximal value of Coulomb interaction energy is equal to the measured maximal value of total kinetic energy of fragments. The pre-scission kinetic energy and intrinsic excitation energy of fragments are assumed to be null. These results suggest that fission process take time to explore all energetically permitted scission configurations.

REFERENCES

[7]. The simulated final mass yield curve $Y(m)$ and the primary mass yield curve